Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits **all the marks** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Paper 1

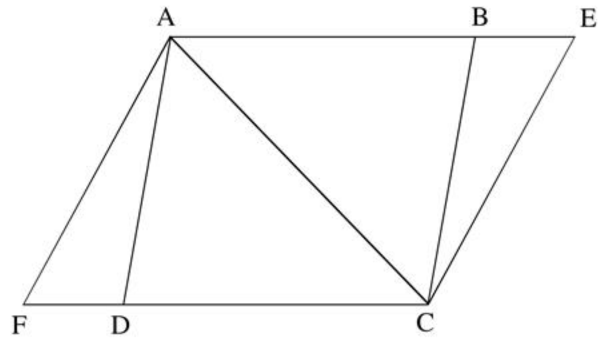
Solution	Marks	Remarks
1. $\dfrac{(x^4y^{-3})^2}{x^{-4}y^7}$		
$=\dfrac{x^8y^{-6}}{x^{-4}y^7}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
$=\dfrac{x^{8+4}}{y^{7+6}}$	1M	for $\dfrac{1}{c^{-p}}=c^p$ or $\dfrac{c^p}{c^q}=c^{p-q}$
$=\dfrac{x^{12}}{y^{13}}$	1A	or $c^{-p}=\dfrac{1}{c^p}$ or $\dfrac{c^p}{c^q}=\dfrac{1}{c^{q-p}}$
	--------(3)	
2. $t(2s-r)=4(s-5t)$		
$2ts-tr=4s-20t$	1M	for expanding one side correctly
$2ts-4s=tr-20t$	1M	for putting s on one side
$(2t-4)s=tr-20t$		
$s=\dfrac{tr-20t}{2t-4}$	1A	for $s=\dfrac{20t-tr}{4-2t}$ or equivalent
	--------(3)	
3. (a) $2p^2+pq-6q^2$		
$=(2p-3q)(p+2q)$	1A	
(b) $2p^2+pq-6q^2+9q-6p$		
$=(2p-3q)(p+2q)+3(3q-2p)$	1M	for using the result of (a)
$=(2p-3q)(p+2q-3)$	1A	or equivalent
	--------(3)	
4. (a) The price of the toy for Andy to purchase it		
$=\$28\div 20\%$	1M	
$=\$140$	1A	
(b) The price of the toy for Calvin to purchase it		
$=\$(140+28)\times(1-25\%)$	1M	for $\$P\times(1-25\%)$
$=\$168\times 75\%$		
$=\$126$	1A	
	--------(4)	

	Solution	Marks	Remarks
5.	Let $5k$ and $4k$ be the numbers of girls and boys respectively, where k is a non-zero constants.	1A	
	Then, $5k+72=2(4k)$	1M	
	$3k=72$		
	$k=24$	1A	
	There are 120 girls and 96 boys.		
	The difference of the number of girls and the number of boys		
	$=120-96$		
	$=24$	1A	
	Let n be the number of girls, then the number of boys is $\frac{4}{5}n$.	1A	
	$n+72=2(\frac{4}{5}n)$	1M	
	$\frac{3}{5}n=72$		
	$n=120$	1A	
	The difference of the number of girls and the number of boys		
	$=120-\frac{4}{5}\times 120$		or $120\times\frac{1}{5}$
	$=24$	1A	
	Let n be the number of boys, then the number of girls is $\frac{5}{4}n$.	1A	
	$\frac{5}{4}n+72=2n$	1M	
	$\frac{3}{4}n=72$		
	$n=96$	1A	
	The difference of the number of girls and the number of boys		
	$=96\times\frac{5}{4}-96$		or $96\times\frac{1}{4}$
	$=24$	1A	
		---------(4)	
6.	(a) $\frac{1-4x}{2}\geq 9$		
	$1-4x\geq 18$		
	$-4x\geq 17$	1M	
	$x\leq -\frac{17}{4}$	1A	
	From $5-x<0$, $x>5$		
	Thus, the solution of (*) is $x\leq -\frac{17}{4}$ or $x>5$.	1A	or equivalent
	(b) -5	1A	
		---------(4)	

	Solution	Marks	Remarks
7. (a)	The coordinates of P' are $(5, 4)$.	1A	or $P' = (5, 4)$ or $P'(5, 4)$
(b)	The coordinates of Q' are $(4-k, -8)$.	1A	or $Q' = (4-k, -8)$ or $Q'(4-k, -8)$
	Since $P'OQ'$ is a straight line ,		
	$\frac{4-0}{5-0} = \frac{-8-0}{4-k-0}$	1M	or equivalent
	$4(4-k) = -40$		
	$4k = 56$		
	$k = 14$	1A	
	The coordinates of Q' are $(4-k, -8)$.	1A	
	Since $P'OQ'$ is a straight line ,		
	$\sqrt{(5-0)^2+(4-0)^2} + \sqrt{(4-k-0)^2+(-8-0)^2} = \sqrt{(5-4+k)^2+(4+8)^2}$	1M	or equivalent
	$\sqrt{41} + \sqrt{k^2-8k+80} = \sqrt{145+2k+k^2}$		
	$41 + 2\sqrt{41}\cdot\sqrt{k^2-8k+80} + k^2 - 8k + 80 = 145 + 2k + k^2$		
	$2\sqrt{41}\cdot\sqrt{k^2-8k+80} = 10k + 24$		
	$164(k^2-8k+80) = 100k^2 + 480k + 576$		
	$64k^2 - 1792k + 12544 = 0$		
	$k^2 - 28k + 196 = 0$		
	$k = 14$	1A	
		---------(4)	
8. (a)	The sum of the scores of the 18 students is 1266 .		
	$50 + a + 80 + b = 70.2 \times 20 - 1266$	1M	or equivalent
	$a + b = 8 \quad \cdots\cdots(1)$		
	And $80 + b - (50 + a) = 34$	1A	or $b - a + 30 = 34$
	$b - a = 4 \quad \cdots\cdots(2)$		
	Solving (1) and (2) , we get $a = 2$ and $b = 6$.	1A	for both correct
(b)	The required probability $= \frac{6}{20}$	1M	
	$= \frac{3}{10}$	1A	
		---------(5)	

Solution	Marks	Remarks

9.



(a) $\because$ $AB = CD$ (opp. sides of parallelogram)

$BE = DF$ (given)

$\therefore$ $AB + BE = CD + DF$

i.e. $AE = CF$

In ΔACE and ΔCAF ,

$\because$ $AE = CF$ (proved)

$AC = CA$ (common side)

$\angle EAC = \angle FCA$ (alt. $\angle$ s, $AB // DC$)

$\therefore$ $\Delta ACE \cong \Delta CAF$ (SAS)

Marking Scheme :	
Case 1 Any correct proof with correct reasons .	3
Case 2 Any correct proof without reasons .	2
Case 3 Incomplete proof with any one correct step and one reason .	1

(b) Form (a) , $CE = AF = 20\,\text{cm}$

$\because$ $\angle ACB = \angle ABC$

$\therefore$ $AB = AC = 15\,\text{cm}$

$AE = 15\,\text{cm} + 10\,\text{cm} = 25\,\text{cm}$

$\because$ $AC^2 + CE^2 = 15^2 + 20^2$

$= 625 = AE^2$

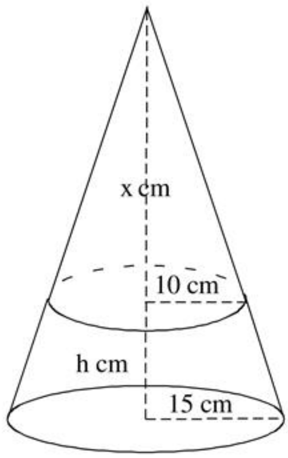
$\therefore$ $\angle ACE = 90°$	1M	f.t.
The area of ΔACE $= \dfrac{15 \times 20}{2} = 150\,(\text{cm}^2)$	1A	

The semi-perimeter of ΔACE $= \dfrac{15 + 25 + 20}{2} = 30\,(\text{cm})$		
The area of ΔACE		
$= \sqrt{30(30-15)(30-25)(30-20)}$	1M	
$= 150\,(\text{cm}^2)$	1A	

---------(5)

		Solution	Marks	Remarks
10.	(a)	Let $C = a + bn$, where a and b are non-zero constants .	1A	
		Sub. $n = 4000$ and $C = 152000$,		
		we have $152000 = a + 4000b$ ------(1)	1M	for either substitution
		Sub. $n = 6000$ and $C = 222000$,		
		we have $222000 = a + 6000b$ ------(2)		
		Solving (1) and (2) , we get $a = 12000$ and $b = 35$.	1A	for both correct
		Let m be the number of books that are published ,		
		then $\frac{12000 + 35m}{m} = 40$		
		$m = 2400$	1A	
		Thus , 2400 books are published .		
			---------(4)	
	(b)	Total publishing cost $= \$(12000 + 35 \times 5000)$		
		$= \$187000$		
		Total income when all the published books are sold		
		$= \$42 \times 5000$		
		$= \$210000$		
		$> \$187000$	1M	
		Thus , the claim is disagreed .	1A	f.t.
		The publishing cost per book		
		$= \$(\frac{12000 + 35 \times 5000}{5000})$		
		$= \$37.4$		
		$< \$42$	1M	
		Thus , the claim is disagreed .	1A	f.t.
			---------(2)	
11.	(a)	The coordinates of point X $= (6,8)$	1A	or $X = (6,8)$ or $X(6,8)$
		The radius of C $= 13$	1A	accept $r = 13$
			---------(2)	
	(b)	$L: 3x - 4y - 11 = 0$		
		The slope of L $= \frac{3}{4}$		accept $m_L = \frac{3}{4}$ or $m = \frac{3}{4}$
		The slope of Γ $= -\frac{4}{3}$		accept $m_\Gamma = -\frac{4}{3}$
		Note that , Γ passes through the centre $X(6,8)$.		
		The equation of Γ is $\frac{y-8}{x-6} = -\frac{4}{3}$	1M	or equivalent
		i.e. $4x + 3y - 48 = 0$		
		$H = (12,0)$, $K = (0,16)$	1A	for both correct
		The area of ΔOHK $= \frac{12 \times 16}{2} = 96$		
		$\frac{1}{4}$ of the area of circle C $= \frac{1}{4} \times \pi \times 13^2$		
		≈ 132.7322896		
		> 96	1M	
		Thus , the claim is correct .	1A	
			---------(4)	

		Solution	Marks	Remarks
12.	(a)	Note that the median is 2.5 ,		
		$9+a=b+c+5$	1M	
		$a=b+c-4 \quad \cdots\cdots(*)$		
		Also note that $a+b+9=28$	1A	
		$b=19-a$		
		Sub. into $(*)$, we have $2a=15+c$		
		But $a>10$, $3<c<8$ and a , b and c are integers .		
		Thus , $a=11$, $b=8$ and $c=7$.	1A	
			---------(3)	
	(b)	The original mean		
		$=\frac{1\times 9+2\times 11+3\times 8+4\times 7+5\times 5}{9+11+8+7+5}$		
		$=2.7$	1A	
		When the numbers of group members of these two groups are 2 and 3 ,	1M	either one
		the least value of the standard deviation ≈ 1.299965114		
		≈ 1.30	1A	
		When the numbers of group members of these two groups are 1 and 5 ,		
		the greatest value of the standard deviation ≈ 1.367753011		
		≈ 1.37	1A	
			---------(4)	

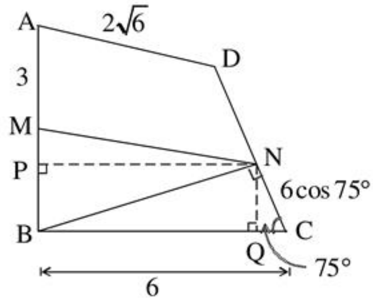
Solution	Marks	Remarks
13. 		
(a) Let h cm be the height of the frustum and x cm be the height of the removed upper cone of the frustum .		
$\frac{x}{x+h}=\frac{10}{15}$	1M	or $\frac{x}{h}=\frac{10}{15-10}$
$15x=10x+10h$		
$5x=10h$		
$x=2h$		
Since the capacities of the cylinder and the frustum are the same , we have		
$\pi\cdot 10^2(31-h)=\frac{1}{3}\pi\cdot 15^2(3h)-\frac{1}{3}\pi\cdot 10^2(2h)$	1M	
$3100-100h=\frac{1}{3}(675h-200h)$		
$9300-300h=475h$		
$h=12$	1A	
The capacity of the frustum		
$=\pi\cdot 10^2(31-h)$		
$=1900\pi\ (cm^3)$	1A	r.t. $5970\ cm^3$
The capacity of the frustum		
$=\frac{1}{3}\pi\cdot 15^2(36)-\frac{1}{3}\pi\cdot 10^2(24)$		
$=1900\pi\ (cm^3)$	1A	r.t. $5970\ cm^3$
	---------(4)	
(b) Let H cm be the depth of water in the cylinder .		
$0.007\times 10^6-1900\pi=100\pi H$	1M	or $0.007\times 10^6-5969.026042$ $=314.1592654H$
$H\approx 3.281692033$		
Depth of water $\approx 3.281692033+12$		
$=15.281692033$		
<15.5	1M	
Thus , the claim is incorrect .	1A	
	---------(3)	

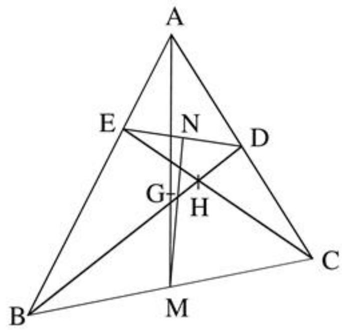
		Solution	Marks	Remarks
14.	(a)	$\because\ p(-2) = p(3) = 0$		
		$\therefore\ x+2$ and $x-3$ are the factor of $p(x)$.	1A	
		Since $p(x)$ is a polynomial with the degree of 3 ,		
		Let $p(x) = (x+2)(x-3)(ax+b)$, where a and b are non-zero constants .	1M+1A	
		$p(1) = -6(a+b) = -18$	1M	either one
		$a+b=3 \quad \cdots\cdots(1)$		
		$p(2) = -4(2a+b) = -20$		either one
		$2a+b=5 \quad \cdots\cdots(2)$		
		Solving (1) and (2) , we get $a=2$ and $b=1$	1A	for both correct
		$\therefore\ p(x) = (x+2)(x-3)(2x+1)$		$p(x) = 2x^3 - x^2 - 13x - 6$
		Let $p(x) = ax^3 + bx^2 + cx + d$, where a , b , c and d are constants .		
		$p(-2) = -8a + 4b - 2c + d = 0$	1M+1A	either one
		$p(3) = 27a + 9b + 3c + d = 0$		
		$p(1) = a + b + c + d = -18$		
		$p(2) = 8a + 4b + 2c + d = -20$		
		Solving these four equations , we get		
		$a=2$, $b=-1$, $c=-13$ and $d=-6$	1M+1A+1A	1M for eliminating any one unknown 1A for any one term correct 1A for all correct
			---------(5)	
	(b)	$p(x) = 3x-9$		
		$(x+2)(x-3)(2x+1) = 3(x-3)$		
		$(x-3)[(x+2)(2x+1)-3] = 0$	1M	for common factor $(x-3)$
		$(x-3)(2x^2+5x-1) = 0$	1A	
		$\therefore\ x=3$ or $x = \dfrac{-5 \pm \sqrt{33}}{4}$	1A	
		Note that $x = \dfrac{-5 \pm \sqrt{33}}{4}$ are not rational numbers.		
		Thus , there is only one rational root of the equation $p(x) = 3x-9$.	1A	f.t.
		$p(x) = 3x-9$		
		$2x^3 - x^2 - 13x - 6 = 3x - 9$	1A	
		$2x^3 - x^2 - 16x + 3 = 0$		
		$(x-3)(2x^2+5x-1) = 0$	1M	for factor $x-3$
		$\therefore\ x=3$ or $x = \dfrac{-5 \pm \sqrt{33}}{4}$	1A	
		Note that $x = \dfrac{-5 \pm \sqrt{33}}{4}$ are not rational numbers.		
		Thus , there is only one rational root of the equation $p(x) = 3x-9$.	1A	f.t.
			---------(4)	

	Solution	Marks	Remarks
15.	The new mean $=38\times 110\%+8=49.8$ (marks)		
	The new standard deviation $=10\times 110\%=11$ (marks)		
	Let x marks be the original score of Kelly .		
	The new standard score of Kelly		
	$=\dfrac{x(110\%)+8-49.8}{11}$	1M	
	$=\dfrac{1.1x-41.8}{11}$		
	$=\dfrac{x-38}{10}$		
	$=-0.1$	1A	
	<0		
	Thus , the claim is disagreed .	1A	
	Let x marks be the original score of Kelly ,		
	then $\dfrac{x-38}{10}=-0.1$		
	$x=37$		
	The new standard score of Kelly		
	$=\dfrac{37(110\%)+8-49.8}{11}$	1M	
	$=-0.1$	1A	
	<0		
	Thus , the claim is disagreed .	1A	
		---------(3)	
16.	(a) The required probability		
	$=\dfrac{C_1^4C_1^4C_5^5+C_2^4C_2^4C_3^5+C_3^4C_3^4C_1^5}{C_7^{13}}$	1M	at least two terms correct in numerator
	$=\dfrac{38}{143}$	1A	r.t. 0.266
	The required probability		
	$=\dfrac{4}{13}\times\dfrac{4}{12}\times\dfrac{5}{11}\times\dfrac{4}{10}\times\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\times P_2^7$		
	$+\dfrac{4}{13}\times\dfrac{3}{12}\times\dfrac{4}{11}\times\dfrac{3}{10}\times\dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{3}{7}\times\dfrac{P_4^7}{2\times 2}$	1M	for any one term correct
	$+\dfrac{4}{13}\times\dfrac{3}{12}\times\dfrac{2}{11}\times\dfrac{4}{10}\times\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{5}{7}\times\dfrac{P_6^7}{6\times 6}$		
	$=\dfrac{38}{143}$	1A	r.t. 0.266
		---------(2)	
	(b) The required probability		
	$=\dfrac{C_1^4C_1^4C_5^5+C_2^4C_2^4C_3^5}{C_1^4C_1^4C_5^5+C_2^4C_2^4C_3^5+C_3^4C_3^4C_1^5}$	1M	for correct numerator or denominator
	$=\dfrac{47}{57}$	1A	r.t. 0.825
		---------(2)	

		Solution	Marks	Remarks
17.	(a)	Let r be the common ratio of the sequence .		
		Then $8r^{6-1} = 1944$	1M	
		$r^5 = 243$		
		$r = 3$	1A	
		$\therefore$ The common ratio of the sequence $= 3$		
		the common ratio of the sequence		
		$= \sqrt[5]{\dfrac{1944}{8}}$	1M	
		$= 3$	1A	
			---------(2)	
	(b)	$\dfrac{8(3^n - 1)}{3-1} > 100000000$	1M	
		$3^n > 25000001$		
		$n\log 3 > \log 25000001$	1M	
		$n > \dfrac{\log 25000001}{\log 3}$		
		≈ 15.50536672		
		$\therefore$ The least value of n is 16 .	1A	
			---------(3)	
18.	(a)	$f(x) = -\dfrac{1}{2}x^2 + \dfrac{1}{4}x + 1$		
		$= -\dfrac{1}{2}(x^2 - \dfrac{1}{2}x) + 1$		
		$= -\dfrac{1}{2}(x^2 - \dfrac{1}{2}x + \dfrac{1}{16} - \dfrac{1}{16}) + 1$	1M	
		$= -\dfrac{1}{2}(x - \dfrac{1}{4})^2 + \dfrac{33}{32}$		
		The coordinates of the vertex of the graph are $(\dfrac{1}{4}, \dfrac{33}{32})$.	1A	
			--------(2)	
	(b)	$-\dfrac{1}{2}x^2 + \dfrac{1}{4}x + 1 = c$		
		$2x^2 - x + 4c - 4 = 0$		
		$x = \dfrac{1 \pm \sqrt{1 - 8(4c-4)}}{4} = \dfrac{1 \pm \sqrt{33 - 32c}}{4}$		
		$PQ = \dfrac{1 + \sqrt{33-32c}}{4} - \dfrac{1 - \sqrt{33-32c}}{4}$	1M	
		$= \dfrac{\sqrt{33-32c}}{2}$		
		$\therefore \dfrac{\sqrt{33-32c}}{2} = \dfrac{1}{2}c$	1M	
		$33 - 32c = c^2$		
		$c^2 + 32c - 33 = 0$		
		$(c+33)(c-1) = 0$		
		$c = -33$ (rejected) or $c = 1$	1A	
			--------(3)	

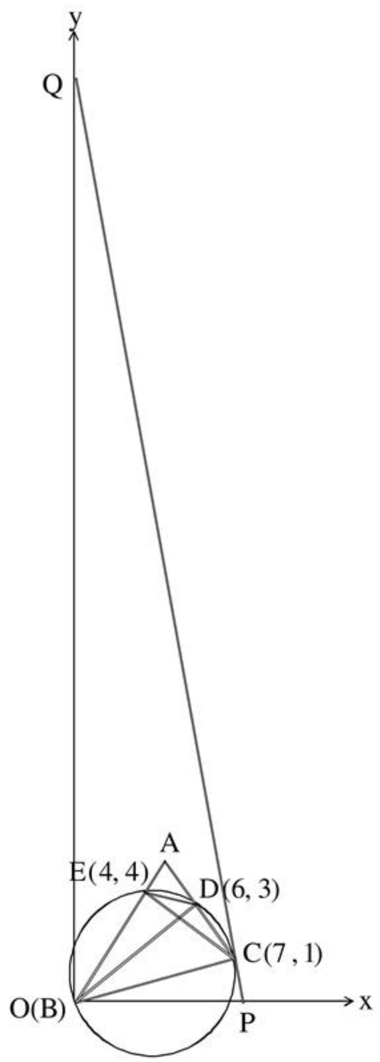
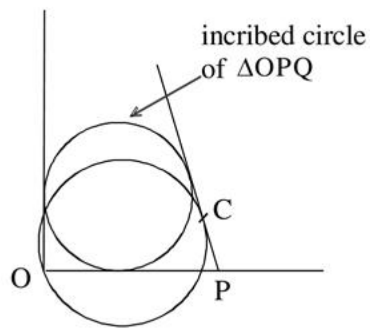
Solution	Marks	Remarks
19. (a) $\angle ADC = 360° - 90° - 2 \times 75° = 120°$		
In ΔADC ,		
$AC^2 = (2\sqrt{6})^2 + (2\sqrt{6})^2 - 2(2\sqrt{6})(2\sqrt{6})\cos 120°$	1M	or $AC = 2(2\sqrt{6}\sin 60°) = 6\sqrt{2}$
$AC^2 = 72$		
In rt.ΔABC ,		
$2AB^2 = AC^2$		
$AB^2 = 36$		
$AB = 6$ cm	1A	
	--------(2)	
(b) (i) Let M be the mid-point of AB .		
$VM = 6\sin 60°$		
$= 3\sqrt{3}$		
$MA = 3$		
In ΔAMD ,		
$MD^2 = 3^2 + (2\sqrt{6})^2 - 2(3)(2\sqrt{6})\cos 75°$	1M	
$= 33 - 12\sqrt{6}\cos 75°$		$MD^2 \approx 25.39230485$
In rt.ΔVMD ,		
$VD^2 = VM^2 + MD^2$		
$= (3\sqrt{3})^2 + 33 - 12\sqrt{6}\cos 75°$		
$VD \approx 7.238252886$		
≈ 7.24 cm	1A	
(ii) $CN = 6\cos 75°$		
≈ 1.552914271		
$VC = \sqrt{6^2 + 6^2} = 6\sqrt{2}$		
Let N' be the foot of the perpendicular from V to CD .		
In ΔVCD ,		
$\cos\angle VCD \approx \dfrac{(6\sqrt{2})^2 + (2\sqrt{6})^2 - 7.238252886^2}{2(6\sqrt{2})(2\sqrt{6})}$	1M	
≈ 0.524519052		
$CN' = 6\sqrt{2}\cos\angle VCD$		
$\approx 6\sqrt{2}(0.524519052)$	1M	
≈ 4.450691742		
$\therefore$ $CN \neq CN'$		
$\therefore$ N is not the foot of the perpendicular from V to CD .		
That is $\angle VNB$ is not the angle between the face VCD and the face $ABCD$.		
Thus , the claim is incorrect .	1A	

Solution	Marks	Remarks
Construct $NP \perp AB$ and $NQ \perp BC$. $PN = BC - QC$ $= 6 - 6\cos 75° \cos 75°$ $MP = MB - PB$ $= 3 - 6\cos 75° \sin 75°$ In rt.ΔMPN, $MN = \sqrt{PN^2 + MP^2}$ $= \sqrt{(6 - 6\cos 75° \cos 75°)^2 + (3 - 6\cos 75° \sin 75°)^2}$ ≈ 5.795554958 In rt.ΔVMN, $VN = \sqrt{VM^2 + MN^2}$ $= \sqrt{(3\sqrt{3})^2 + 5.795554958^2}$ ≈ 7.783858765 In ΔVNC, $VN^2 + NC^2 = 7.783858765^2 + (6\cos 75°)^2$ ≈ 63.00000001 Also, $VC^2 = (6\sqrt{2})^2 = 72$ $\therefore$ $VN^2 + NC^2 \neq VC^2$ $\therefore$ $\angle VNC$ is not a right angle. That is $\angle VNB$ is not the angle between the face VCD and the face ABCD. Thus, the claim is incorrect.	1M 1M 1A ---------(5)	

Solution	Marks	Remarks
20. (a) (i) $\because$ H is the orthocenter		
$\therefore \angle BDC = \angle BEC = 90^\circ$		
$\therefore$ B , C , D and E are concyclic		
(converse of $\angle$ s in the same segment)		
$\because \angle BDC = 90^\circ$		
$\therefore$ BC is a diameter of the circle		
(converse of $\angle$ in semi-circle)		
Since G is the centroid of ΔABC ,		
AM is the median of BC .		
That is M is the mid-point of BC .		
Thus , M is the centre of the circle BCDE .		

Marking Scheme :	
Case 1 Any correct proof with correct reasons .	3
Case 2 Any correct proof without reasons .	2
Case 3 Incomplete proof with any one correct step and one reason .	1

Solution	Marks	Remarks
(ii) $\because$ ME and MD are radii of the circle		
$\therefore ME = MD$		
$\therefore \Delta MED$ is an isosceles triangle		
But N is the mid-point of the base ED		
$\therefore MN \perp ED$		
Thus , the claim is correct .	1A	
	---------(4)	
(b) (i) The slope of ED $= \frac{3-4}{6-4} = -\frac{1}{2}$		
The slope of MN $= 2$		
And $N = (5 , \frac{7}{2})$		
The equation of MN is $\frac{y-\frac{7}{2}}{x-5} = 2$	1M	
i.e. $4x-2y-13=0$		
Solving with $x-7y=0$,		
we get $M = (\frac{7}{2} , \frac{1}{2})$		
$MD = \sqrt{(6-\frac{7}{2})^2 + (3-\frac{1}{2})^2}$		or $ME = \sqrt{(4-\frac{7}{2})^2 + (4-\frac{1}{2})^2}$
$= \frac{5\sqrt{2}}{2}$		$= \frac{5\sqrt{2}}{2}$
The equation of the circle BCDE is		
$(x-\frac{7}{2})^2 + (y-\frac{1}{2})^2 = \frac{25}{2}$	1M	
i.e. $x^2 + y^2 - 7x - y = 0$		
sub. $x = 7y$		
$49y^2 + y^2 - 49y - y = 0$		
$y^2 - y = 0$		
$y = 0$ (rejected) or $y = 1$		
$\therefore$ The coordinates of point C are $(7,1)$.	1A	

Solution	Marks	Remarks
(ii) The slope of the tangent $= -7$		
The equation of the tangent is $\frac{y-1}{x-7} = -7$	1M	
i.e. $7x + y - 50 = 0$		
The coordinates of points P and Q are $(\frac{50}{7}, 0)$ and $(0, 50)$ respectively		
$PQ = \sqrt{(\frac{50}{7} - 0)^2 + (0 - 50)^2}$		
$= \frac{250\sqrt{2}}{7}$		
Let r be the radius of the inscribed circle of ΔOPQ .		
Then $\frac{1}{2}(\frac{50}{7} + 50 + \frac{250\sqrt{2}}{7})r = \frac{1}{2} \times \frac{50}{7} \times 50$	1M+1A	
$r = \frac{2500}{400 + 250\sqrt{2}}$		
$= \frac{50}{8 + 5\sqrt{2}}$ $(= \frac{200 - 125\sqrt{2}}{7})$		
≈ 3.317614958		
≈ 3.32	1A	
$\therefore$ The radius of the inscribed circle of ΔOPQ is 3.32.		
	---------(7)	

Paper 2

1. [B]

$$(-3)^{2017}(\frac{1}{9})^{1009} = (-3)^{2017}(\frac{1}{3^2})^{1009}$$

$$= -3^{2017} \cdot \frac{1}{3^{2018}}$$

$$= -\frac{1}{3}$$

2. [C]

$$(x-2)(x^2-2x+4) = x^3-2x^2+4x-2x^2+4x-8$$

$$= x^3-4x^2+8x-8$$

3. [B]

$2m+n+1=-1$ ------ (1)

$m-2n+5=-1$ ------ (2)

(2) × 2, $2m-4n+10=-2$ ------ (3)

(1) – (3), we have $5n-9=1$, $n=2$

sub. into (2), we have $m=-2$

$\therefore$ $m+n=0$

4. [C]

$0.74496 = 0.745$ (correct to 3 significant figures)

$0.74505 = 0.745$ (correct to 3 significant figures)

$\therefore$ $x = 0.745$ (correct to 3 significant figures)

5. [D]

$(x+2)^2 + p \equiv (x-1)(x+q)+3$

$x^2+4x+4+p \equiv x^2+(q-1)x+3-q$

$\therefore$ $q-1=4$ and $4+p=3-q$

$\therefore$ $q=5$, $p=-6$

Alternative solution:

sub. $x=1$, we have $9+p=3$, $p=-6$

sub. $x=0$, we have $4-6=-q+3$, $q=5$

6. [B]

$$\begin{cases} -2x+5<13 \\ 13<5x-2 \end{cases}$$

$$\begin{cases} -2x<8 \\ 15<5x \end{cases}$$

$$\begin{cases} x>-4 \\ x>3 \end{cases}$$

$\therefore\ x>3$

7. [A]

$\because\ x=-1$ is a root of the equation $2x^2-x+k=0$

$\therefore\ 2+1+k=0,\ k=-3$

$\because\ \beta$ is a root of the equation $2x^2-x-3=0$

$\therefore\ 2\beta^2-\beta-3=0$

$-4\beta^2+2\beta+6=0$

$11+2\beta-4\beta^2=5$

Alternative solution:

$-1+\beta=\frac{1}{2},\ \beta=\frac{3}{2}$

$11+2\beta-4\beta^2=11+3-9=5$

8. [A]

The graph opens upwards, thus $p>0$.

The x-coordinates of the vertex $=-\frac{q}{2p}<0$ (the vertex lies in the third quadrant)

Since $p>0,\ q>0$.

9. [D]

Let the weight of Clara be 100 units,

then the weight of Sunny is $100\times120\%=120$ units

and the weight of Kenny is $120\div(1-20\%)=150$ units

$\therefore$ Kenny is 50% heavier than Clara.

10. [A]

Compound interest $=50000\times(1+1.2\%)^6-50000=3710$

Simple interest $=50000\times2.5\%\times3=3750$

Difference between two interests obtained $=\$(3750-3710)=\$40$

11. [D]

$$\frac{1}{2}a = 2b = 3c$$

$$\frac{a}{12} = \frac{b}{3} = \frac{c}{2}$$

$\therefore$ $a : b : c = 12 : 3 : 2$

$$\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = \frac{1}{12} : \frac{1}{3} : \frac{1}{2} = 1 : 4 : 6$$

Alternative solution:

$$\frac{1}{2}a = 2b = 3c$$

$$\frac{2}{a} = \frac{1}{2b} = \frac{1}{3c}$$

$$\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = \frac{1}{2} : \frac{2}{1} : \frac{3}{1} = 1 : 4 : 6$$

12. [C]

$z = \frac{kx^2}{y}$, where k is a non-zero constant.

$$x_1 = \frac{6}{5}x, \quad y_1 = \frac{3}{4}y$$

$$z_1 = \frac{k(\frac{6}{5}x)^2}{\frac{3}{4}y} = \frac{48}{25}(\frac{kx^2}{y}) = \frac{48}{25}z$$

$\therefore$ z is increased by $\frac{23}{25} \times 100\% = 92\%$.

13. [C]

Number of dots in the 6th pattern $= 3 + 4 + 5 + 6 + 7 + 8 = 33$

Alternative solution:

Number of dots in the 6th pattern $= 49 - (1 + 2 + 3 + 4 + 6) = 33$

14. [A]

The weight of the bag of salt ≥ 7.5 kg = 7500 g

The weight of each packet of salt < 15.5 g

while $\frac{7500}{15.5} = 483.87$

$\therefore$ The least possible value of n is 483.

15. [D]

$\angle BCG = 108° - 90° = 18°$

$\because$ $CB = CG$

$\therefore \angle CBG = \dfrac{180° - 18°}{2} = 81°$

$\angle ABG = 108° - 81° = 27°$

16. [B]

Let $AE = EC = x$ cm , $AF = y$ cm

then $x^2 + (y+3)^2 = 10^2$ $(DE = 10\text{ cm})$

$x^2 + y^2 + 6y = 91$ ------ (1)

Also $(2x)^2 + y^2 = 13^2$ $(CF = 13\text{ cm})$

$x^2 = \dfrac{169 - y^2}{4}$ ------ (2)

sub. (2) into (1), we have $\dfrac{169 - y^2}{4} + y^2 + 6y = 91$

$3y^2 + 24y - 195 = 0$

$y^2 + 8y - 65 = 0$

$y = 5$ or $y = -13$ (rejected)

$x = \sqrt{\dfrac{169 - 25}{4}} = 6$,

$BC = \sqrt{20^2 - 12^2} = 16$,

Area of ΔABC $= \dfrac{12 \times 16}{2} = 96\ (\text{cm}^2)$

17. [A]

Let r cm be the base radius of the circular cone,

then $2\pi \times 10 \times \dfrac{216}{360} = 2\pi r$

$r = 6$

The height of the circular cone $= \sqrt{10^2 - 6^2} = 8$

The volume of the circular cone $= \dfrac{1}{3}\pi \times 6^2 \times 8 = 96\pi(\text{cm}^3)$

18. [D]

Area of ΔABE $= 96 \times \dfrac{3^2}{2^2} = 216$

Area of ΔDEC $= 216 \times \dfrac{2}{3} = 144$

Area of $\Delta ADE = 216 + 144 = 360(cm^2)$

19. [D]

$$AC = \frac{DC}{\cos\beta}$$

$$DB = \frac{DC}{\tan\alpha}$$

$$\therefore \frac{AC}{DB} = \frac{\tan\alpha}{\cos\beta}$$

20. [C]

$$\frac{\cos 0° + \cos(90° - \theta)}{\sin(90° + \theta)} - \frac{\cos(180° + \theta)}{1 - \sin(360° - \theta)}$$

$$= \frac{1 + \sin\theta}{\cos\theta} - \frac{-\cos\theta}{1 + \sin\theta}$$

$$= \frac{(1 + \sin\theta)^2 + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{2(1 + \sin\theta)}{\cos\theta(1 + \sin\theta)} \quad = \frac{2}{\cos\theta}$$

21. [B]

Join OD, AC and AD.

$\angle DOC = 180° - 2 \times 46° = 88°$

$$\angle DAC = \frac{88°}{2} = 44°$$

$\angle ADC = 180° - 123° = 57°$

$\angle ACD = 180° - 57° - 44° = 79°$

$\angle AED = 180° - 79° = 101°$

Alternative solution:

Join AO and AC.

$\angle AOC = 360° - 2 \times 123° = 114°$

$$\angle ACO = \frac{180° - 114°}{2} = 33°$$

$\angle AED = 180° - 33° - 46° = 101°$

22. [C]

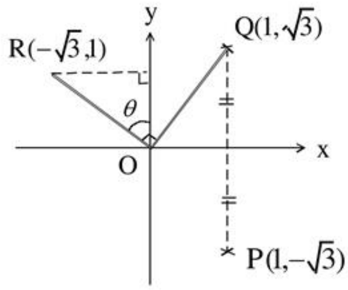
$(n - 2) \times 180° = 1440°$ (n be the number of sides)

$n = 10$

Each interior angle $= \frac{1440°}{10} = 144°$

Number of diagonals $= \dfrac{10(10-3)}{2} = 35$ (or $C_2^{10} - 10 = 35$)

23. [D]



$\tan\theta = \dfrac{\sqrt{3}}{1}$

$\theta = 60°$

$OR = \sqrt{(\sqrt{3})^2 + 1^2} = 2$

$\therefore$ The polar coordinates of point R are $(2, 150°)$.

24. [D]

According to the converse of $\angle$ in semi-circle.

25. [D]

Let $(k, 0)$ be the point of intersection,

then $2k + 4 = 0$, $k = -2$

And $mk + 2 = 0$

$-2m + 2 = 0$, $m = 1$

L_1 : $2x - y + 4 = 0$, slope $m_1 = -\dfrac{2}{-1} = 2$

L_2 : $mx + ny + 2 = 0$, slope $m_2 = -\dfrac{m}{n}$

$\because$ $L_1 \perp L_2$

$\therefore$ $2(-\dfrac{m}{n}) = -1$

sub. $m = 1$, we get $n = 2$

26. [A]

$\dfrac{1}{2}x^2 + \dfrac{1}{2}y^2 - 3x + 5y + 9 = 0$

$x^2 + y^2 - 6x + 10y + 18 = 0$

Centre $= (3, -5)$, radius $= \dfrac{1}{2}\sqrt{36 + 100 - 72} = 4$

Put $x = 0$, we have $y^2 + 10y + 18 = 0$

$\because$ $\Delta = 10^2 - 72 > 0$

$\therefore$ The circle and the y-axis intersect at two distinct points.

The distance between the origin and the centre $= \sqrt{3^2 + 5^2} = \sqrt{34} > 4$

27. [A]

Product	1	4	6	15
1		4	6	15
4	4		24	60
6	6	24		90
15	15	60	90	

The required probability $=\frac{2}{12}=\frac{1}{6}$

Alternative solution:

The required probability $=\frac{1}{C_2^4}=\frac{1}{6}$

28. [C]

The expected value $=20\times\frac{5}{10}+50\times\frac{4}{10}+500\times\frac{1}{10}$

$=80$ (dollars)

29. [B]

When x increases, $\frac{1}{y}$ decreases, thus y increases.

30. [A]

$$\frac{11+18+12+14+14+20+7+16+10+p+q}{11}=14$$

$\therefore\ p+q=32$

When $p=13$ and $q=19$, the median is 14.

Thus, II and III may not be true.

31. [C]

$$\frac{1}{x^2-2x+1}-\frac{1}{x^2-1}$$

$$=\frac{1}{(x-1)^2}-\frac{1}{(x+1)(x-1)}$$

$$=\frac{(x+1)-(x-1)}{(x-1)^2(x+1)}$$

$$=\frac{2}{(x-1)^2(x+1)}$$

32. [A]

Slope $=\frac{4}{2}=2$

$\log_{\frac{1}{2}} y=2x+4$

$y=(\frac{1}{2})^{2x+4}=2^{-2x-4}=2^{-4}\cdot 2^{-2x}=\frac{1}{16}(\frac{1}{4})^x$

$\therefore\ a = \frac{1}{16}$

33. [D]

$$5\times 2^7 + 2^5 + 17 = (2^2+1)\times 2^7 + 2^5 + 2^4 + 1$$
$$= 2^9 + 2^7 + 2^5 + 2^4 + 1$$
$$= 1010110001_2$$

34. [C]

$$uv = \frac{i}{a+i}\cdot\frac{i}{a-i} = \frac{-1}{a^2+1}$$ is a real number

Thus, I is true.

$$u = \frac{i}{a+i}\times\frac{a-i}{a-i} = \frac{1+ai}{a^2+1} = \frac{1}{a^2+1} + \frac{a}{a^2+1}i$$

$$v = \frac{i}{a-i}\times\frac{a+i}{a+i} = \frac{-1+ai}{a^2+1} = -\frac{1}{a^2+1} + \frac{a}{a^2+1}i$$

Thus, II is true.

$$\frac{1}{u} = \frac{a+i}{i} = \frac{ai-1}{-1} = 1-ai$$

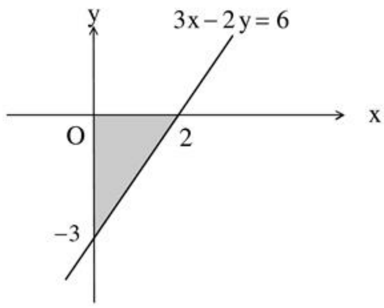
$$\frac{1}{v} = \frac{a-i}{i} = \frac{ai+1}{-1} = -1-ai$$

Thus, III is false

35. [D]

When $\begin{cases} x\ge 0 \\ y\le 0 \\ 3x-2y\le 6 \end{cases}$,

$p = 2x-3y$ have both maximum and minimum values.



36. [B]

$b^2 = ac$

$2\log b = \log a + \log c$

Thus, I is true.

$b^2 = ac$, $\frac{c}{b} = \frac{b}{a}$

a, b, c is a geometric sequence.

$\frac{2^c}{2^b} = 2^{c-b}$, $\frac{2^b}{2^a} = 2^{b-a}$

Thus, II is false.

Also, $\frac{c^m}{b^m} = (\frac{c}{b})^m$, $\frac{b^m}{a^m} = (\frac{b}{a})^m = (\frac{c}{b})^m$

Thus, III is true.

37. [D]

$\sin x(3\cos^2 x+4\cos x-4)=0$

$\sin x(3\cos x-2)(\cos x+2)=0$

$\sin x=0$ has 3 roots, $\cos x=\frac{2}{3}$ has 2 roots, $\cos x=-2$ has no solution.

$\therefore$ The equation has 5 roots.

38. [C]

$a=-\frac{3+1}{2}=-2$, $k=1$

Form $0=-2\cos(40°+\theta)+1$,

$\cos(40°+\theta)=\frac{1}{2}$

$\theta=20°$

39. [D]

$AB=\sqrt{5^2+12^2}=13$

Construct $QN\perp AB$ such that N is the foot of the perpendicular.

$QN=\frac{12\times 5}{13}=\frac{60}{13}$

In rt.ΔPQN,

$\tan\theta=\frac{8}{QN}=\frac{8}{\frac{60}{13}}=\frac{26}{15}$

40. [B]

Join PB. Let $\angle PAQ=x$, $\angle RPB=a$, $\angle RAQ=b$.

$\angle APB=90°$

$x=\angle RPT=44°$

Also, $a=b$

In ΔAPR,

$a+90°+x+b+24°=180°$

$2b=22°$, $b=11°$

$\angle AQP=b+24°=35°$

41. [B]

The slope of the straight line passing through $(3,-1)$ and perpendicular to $3x+4y+5=0$ is $\frac{4}{3}$.

The equation of this straight line is $\frac{y+1}{x-3}=\frac{4}{3}$, i.e. $4x-3y-15=0$

On solving $3x+4y+5=0$ and $4x-3y-15=0$, the point of intersection is $(\frac{9}{5},-\frac{13}{5})$

$$\text{radius}=\sqrt{(\frac{9}{5}-3)^2+(-\frac{13}{5}+1)^2}=2$$

The equation of the circle is $(x-3)^2+(y+1)^2=4$

i.e. $x^2+y^2-6x+2y+6=0$

Alternative solution:

$$\text{Radius}=\left|\frac{3(3)+4(-1)+5}{\sqrt{3^2+4^2}}\right|=2$$

The equation of the circle is $(x-3)^2+(y+1)^2=4$

42. [C]

$$\text{The required probability}=\frac{3}{5}\times\frac{3}{7}\times\frac{3}{5}+\frac{3}{5}\times\frac{4}{7}\times\frac{2}{5}+\frac{2}{5}\times\frac{2}{7}\times\frac{4}{5}+\frac{2}{5}\times\frac{5}{7}\times\frac{3}{5}$$

$$=\frac{97}{175}$$

43. [C]

$$\text{The number of permutations}=5!C_3^6C_2^4\cdot 3!\cdot 2!$$

$$=172800$$

44. [A]

The inter-quartile range $=64-42=22$ marks

The mean $=55$ marks, the standard deviation $=14.6628783$ marks

The standard score of the student who gets 31 marks $=\frac{31-55}{14.6628783}=-1.636786415>-2$

Thus, II is true.

The corresponding score with standard score 1.3 is $1.3\times14.6628783+55=74.06174179$

Only 2 students whose standard scores are above 1.3.

Thus, III is false.

45. [A]

$$\text{Mean}=\frac{-3a+b-3a+5b-3a-3b-3a+9b-3a-7b}{5}$$

$$=-3a+b$$

$$\text{Standard deviation}=\sqrt{\frac{0^2+(4b)^2+(-4b)^2+(8b)^2+(-8b)^2}{5}}$$

$$=\sqrt{\frac{160}{5}}b$$

$$= 4\sqrt{2}b$$

試卷二

Paper 2

題號 Question No.	答案 Key	題號 Question No.	答案 Key
1.	B (74)	26.	A (43)
2.	C (89)	27.	A (56)
3.	B (78)	28.	C (66)
4.	C (51)	29.	B (45)
5.	D (56)	30.	A (34)
6.	B (65)	31.	C (66)
7.	A (63)	32.	A (43)
8.	A (57)	33.	D (51)
9.	D (42)	34.	C (36)
10.	A (46)	35.	D (30)
11.	D (44)	36.	B (39)
12.	C (66)	37.	D (28)
13.	C (85)	38.	C (39)
14.	A (23)	39.	D (39)
15.	D (55)	40.	B (39)
16.	B (56)	41.	B (39)
17.	A (51)	42.	C (36)
18.	D (38)	43.	C (37)
19.	D (63)	44.	A (32)
20.	C (61)	45.	A (32)
21.	B (53)		
22.	C (80)		
23.	D (47)		
24.	D (44)		
25.	D (41)		

註：括號內數字為答對百分率。

Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.